Suitability of stochastic models for mortality projection in Korea: a follow-up discussion

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Abstract

Due to an increased demand for longevity risk analysis, various stochastic models have been suggested to evaluate uncertainty in estimated life expectancy and the associated value of future annuity payments. Recently updated data allow us to analyze mortality for a longer historical period and extended age ranges. This study followed up previous case studies using up-to-date empirical data on Korean mortality and the recently developed R package StMoMo for stochastic mortality models analysis. The suitability of stochastic mortality models, focusing on retirement ages, was investigated with goodness-of-fit, validity of models, and ability of generating reasonable sets of simulation paths of future mortality. Comparisons were made across various types of models. Based on the selected models, the variability of important estimated measures associated with pension, annuity, and reverse mortgage were quantified using simulations.

Keywords: annuity, longevity risk, mortality projection, retirement income, simulation, stochastic mortality models

1. Introduction

With significant increase in life expectancy, securing a certain level of retirement income for an individual has become an important contemporary issue. Similar to other developed countries, the main sources of retirement income in Korea are personal savings in a variety of assets, pension systems corresponding to occupation, individual annuity contracts, reverse mortgage, etc. Since the payment period for pension and annuity depends on the lifetime of a beneficiary, mortality rate is one of the essential actuarial factors used to determine the amount of regular (monthly) payment to a pensioner or an annuitant.

Therefore, forecasting future mortality of the elderly population is a fundamental task for managing pension and annuity providers' longevity risk, which becomes a financial challenge due to longer-than-estimated lifetimes of pensioners or annuitants. There are various methods of forecasting future mortality, and group of stochastic mortality models are particularly popular. Depending on the structure, a model can reflect the age patterns, time trends, and cohort effects of mortality. Since uncertainty in future mortality rates should be appropriately considered in longevity risk management, a model for actuarial application should be carefully selected.

The goal of this study is to evaluate the suitability of each stochastic mortality model for actuarial application in longevity risk management of life annuity and hence, it is desirable to use data containing broad age ranges of the elderly population over several periods to reflect the complex structure

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of historical mortality rates. Previous discussions on stochastic mortality models applied to Korean mortality data used empirical data over a period of less than 30 years, and mortality rates with an upper limit of 79 years before year 2000. Recent mortality data updates by Statistics Korea from 1983 to 2018 allow us to analyze mortality rates with ages up to 99 years. Therefore, the availability of new data containing information on more recent mortality trends and a wider age range presents a timely opportunity to follow up on the previous studies of mortality in Korea.

This study aims to analyze the mortality of the elderly population in Korea using empirical data and evaluating several stochastic models to identify an appropriate model for forecasting mortality rates by retirement age. In addition, as a follow-up to previous case studies on Korean mortality, re-evaluation of stochastic mortality models is necessary to identify the best up-to-date models for longevity risk management. This re-evaluation is necessary since the degree of mortality improvement in the last few years has been relatively small. It is expected that this study will provide information on various aspects of stochastic mortality models and their capability of longevity risk analysis for systems involving life annuity.

The rest of this paper is structured as follows. Section 2 presents a brief review of previous research on stochastic mortality models. Section 3 introduces the historical Korean mortality data analyzed in this study and discusses exploratory data analysis results. Section 4 describes the investigation on the suitability of various stochastic models for analyzing longevity risk using Korean mortality data, while Section 5 illustrates the projection of mortality rates and analysis of life expectancy and associated actuarial value of life annuity based on the selected models. Section 6 presents the conclusions.

2. Literature review

In mortality projection, it is necessary to consider important characteristics such as mortality improvement and cohort effects in addition to increases in mortality rate with age. Comprehensive discussion on many aspects that should be considered in mortality modeling can be found in Booth and Tickle (2008), and Macdonald *et al.* (2018). Although both deterministic and stochastic models can be used for mortality projection, stochastic models are considered to be more suitable for mortality/longevity risk management due to their capacity to reflect the uncertain nature of mortality rates. Since Lee and Carter (1992) suggested a stochastic mortality model (Lee-Carter model) for population projection in the United States, there have been extensive discussions regarding the performance of the model and several extensions of the model have been discussed for actuarial applications.

The rationale and proposed structure of the various stochastic mortality models were discussed in Renshaw and Haberman (2006), Hobcraft *et al.* (1982), Cairns *et al.* (2006, 2009), and Plat (2009). The extensions focused on including terms regarding cohort effects and modification of functional form regarding time trends. To select the appropriate mortality model for a specific population with a certain purpose, it is important to understand the characteristics of each suggested model.

Various case studies can provide insights into the performance aspects of a model. There are case studies comparing stochastic mortality models and analyzing overall mortality rates of the Korean population. Park *et al.* (2005), Jeong and Kim (2011), Kim (2012), Porapol (2013), Song (2014), Yang (2014), and Jho and Lee (2018) analyzed Korean mortality data using stochastic mortality models and compared the performance of the models based on criteria such as goodness-of-fit, robustness, and results of prediction. Since stochastic models are used for mortality projection in the future, the reasonableness of prediction associated with various ages and simulation paths needs to be carefully considered.

Figure 1: Population (left) and mean age (right) by year.

Comparison of stochastic mortality models using population data in other countries has been performed by Cairns *et al.* (2009, 2011), Carfora *et al.* (2017), and Bozikas and Pitselis (2018). In addition, although stochastic mortality models are usually applied to model overall mortality, Mubarik *et al.* (2020) tested their suitability for modeling mortality due to breast cancer. A group of studies such as De Jong and Tickle (2006), Park *et al.* (2013), Lee *et al.* (2016), Santolino (2020), and De Jong *et al.* (2020) suggested methods for improving the Lee-Carter model in terms of parameter estimation, robustness, and interpretability.

There are various tools that facilitate stochastic mortality modeling, including packages based on R, a popular software environment for statistical computing and graphics. Early developments include R packages named “demography” and “ilc”, and an R function called LifeMetrics. Additionally, an R package StMoMo has been developed to provide relatively more comprehensive analytic tools for stochastic mortality modeling. The package has great flexibility for modeling various types of stochastic mortality models and contains many novel useful functions. Features and usages of the StMoMo package are described in Villegas *et al.* (2018).

3. Mortality data

3.1. Data source

Statistics Korea provides various data associated with factors that cause change in the Korean population, including marriage, fertility, mortality, and immigration. For comprehensive mortality analysis, the mortality rate, population size, and number of deaths by age and gender are required. There has been a recent update of relevant information by Statistics Korea that now includes mortality rates from 1970 to 2018. Population by age is available for up to age 79 from 1970 to 2000 and up to age 99 through 2018. In addition, the number of deaths per age is available up to age 99 from 1983 to 2018.

Figure 1 presents population and mean age by year. Population has increased continuously from

Figure 2: Mortality rates of the Korean elderly population by year from 1970 to 2018.

32.1 million in 1970 to 51.6 million in 2018, although population growth has declined over time, mainly due to a decrease in fertility rate. Similarly, the gap in population size between males and females has become narrower in recent years. In addition, the average age of Korea's population has increased sharply from 22.2 to 40.0 for males and from 23.6 to 42.4 for females, implying that aging has progressed rapidly in Korea in recent decades. Therefore, fair valuation of pension and annuity and their risk management will be important issues as time passes.

Longevity risk is associated with survival rates in retirement ages. When the actual survival period is longer than estimated during the payment period of pension and annuity, the amount of pension or annuity payment will be larger than expected, which threatens the financial soundness of pension and annuity providers. Therefore, actuaries should understand the various characteristics of mortality in older ages and carefully derive mortality assumptions with appropriate models. In this study, we focused on the mortality in retirement ages starting age 55 to reflect the recent minimum age revision for public reverse mortgages.

3.2. Empirical mortality

The mortality rates by age for all causes published by Statistics Korea were observed. Figure 2 shows mortality improvement for both males and females for over 40 years. In other words, mortality has decreased gradually for both genders. The degree of mortality improvement varies by age. An improvement in mortality rate is seen from age 55 up to the early eighties after which it decreases. It can be deduced that the increased life expectancy is strongly associated with a decrease in mortality from various adult diseases due to the progress made in medical science and in public policy for disease prevention and its implementation.

Remarkably, the mortality rate between 2017 and 2018 is barely observable. Further, mortality

Figure 3: Mortality rates by year for the Korean population for ages 55, 65, and 75.

rates for some ages in 2018 even increased compared to those in 2017. This implies that mortality trends should be carefully observed since they may follow patterns different from past trends. If the mortality model does not reflect a slowdown in mortality improvement, mortality projection may underestimate future mortality.

For further observation, mortality trend by age was explored. Mortality rate by year for several selected ages is illustrated in Figure 3. Mortality rates by year for ages 55, 65, and 75 show a declining pattern in Figure 2. The gap in mortality rates among the presented ages have dramatically reduced over time. Since it is reasonable to expect that the age-corresponding pattern of mortality due to biological deterioration of the human body would continue, improvements in mortality rate can be said to be constrained by age. In addition, the gap in mortality rates between males and females reduced over time.

4. Stochastic mortality models

4.1. Models

As reviewed in Section 2, various mathematical forms to capture the stochastic nature of mortality rates over time have been suggested. Stochastic mortality models basically formularize mortality rates or central death rates for each age and year, denoted by $q_{x,t}$ and $m_{x,t}$, respectively. The mortality rate $q_{x,t}$ is the probability that a person aged x in year t dies within a year. Central death rate $m_{x,t}$ is the weighted average of forces of mortality experienced next year by a group of survivors aged x , which is usually approximated by $m_{x,t} \approx -\log(1 - q_{x,t})$ assuming constant force of mortality in fractional ages. The mathematical formulation of the models considered in this study following Villegas *et al.* (2018) is as below.

- Lee-Carter (LC) model

$$\log(m_{x,t}) = \alpha_x + \beta_x \cdot \kappa_t + \varepsilon_{x,t}.$$

- Cairn-Blake-Dowd (CBD) model

$$\log\left(\frac{q_{x,t}}{1 - q_{x,t}}\right) = \kappa_t^{(1)} + (x - \bar{x}) \cdot \kappa_t^{(2)} + \varepsilon_{x,t}.$$

- Age-Period-Cohort (APC) model

$$\log(m_{x,t}) = \alpha_x + \kappa_t + \gamma_{t-x} + \varepsilon_{x,t}.$$

- Renshaw and Haberman (RH) model

$$\log(m_{x,t}) = \alpha_x + \beta_x \cdot \kappa_t + \gamma_{t-x} + \varepsilon_{x,t}.$$

- CBD-M7 model

$$\log\left(\frac{q_{x,t}}{1 - q_{x,t}}\right) = \kappa_t^{(1)} + (x - \bar{x}) \cdot \kappa_t^{(2)} + \left((x - \bar{x})^2 - \hat{\sigma}_x^2\right) \cdot \kappa_t^{(3)} + \gamma_{t-x} + \varepsilon_{x,t}.$$

- PLAT model

$$\log\left(\frac{q_{x,t}}{1 - q_{x,t}}\right) = \alpha_x + \kappa_t^{(1)} + (x - \bar{x}) \cdot \kappa_t^{(2)} + \gamma_{t-x} + \varepsilon_{x,t}.$$

In these mathematical expressions, α_x indicates age-increasing patterns of mortality, and k_t terms reflect time trends that will be used to project future mortality using a time series (ARIMA) model. When a term is expressed as a product of κ_t and a term involving age x , it reflects the dependence of the mortality trend on each age as observed in Figure 2. Finally, γ_{t-x} indicates cohort effects, a possible distinct pattern of mortality observed only within a group of people from the same birth year. Finally, $\bar{x}$ is the mean of ages in the data used to construct a model.

In the LC model, mortality level by age and year is quantified by age specific parameter α_x and time-specific parameter κ_t that is multiplied by another age specific parameter β_x to reflect that degree of mortality improvement varies according to age. Forecasting future mortality is based on time series model that fitted to the estimated κ_t values. The RH model is simple extension of the LC model by adding cohort effect term γ_{t-x} . Also, the APC model is a special case of the RH model assuming $\beta_x = 1$ for all age x .

The CBD model has different structure comparing with the LC model and its extensions. In particular, there are two sets of time-specific parameter, one of which is multiplied by the term indicating age. The CBD model does not contain the term for cohort effect. Similar to the LC model, forecasting future mortality is based on the two sets of estimated time-specific parameters using multivariate time series model or two independent series models. The CBD-M7 model is one of several extensions of the CBD model that has additional terms including cohort effect. Finally, the PLAT model has hybrid form of the APC model and the CBD model.

Stochastic mortality models can accommodate age, time, and cohort effect with simple mathematical form. Most importantly, the models can generate sample paths of future mortality rates so that

Table 1: Goodness-of-fit of models

Models	Number of parameters	Males		Females	
		AIC(rank)	BIC(rank)	AIC(rank)	BIC(rank)
LC	124	16,550.91(4)	17,218.37(4)	19,358.54(5)	20,026.00(5)
CBD	72	22,670.82(6)	23,058.38(6)	30,331.46(6)	30,719.02(6)
APC	152	18,332.63(5)	19,150.81(5)	17,710.68(4)	18,528.86(4)
RH	197	15,627.45(2)	16,687.85(2)	16,037.46(3)	17,097.86(3)
CBD-M7	179	15,232.55(1)	16,196.06(1)	15,601.82(1)	16,565.33(1)
PLAT	186	15,722.40(3)	16,723.59(3)	15,980.43(2)	16,981.62(2)

Figure 4: Estimated parameters by models (males).

uncertainty in future change of mortality level can be quantified. Scenario analysis based on the generated sample paths by stochastic mortality model can be useful to measure longevity risk in annuity or pension portfolios. However, a stochastic mortality model can be too simple to explain complicated mortality dynamics. Further, projection of mortality is based on the assumption that patterns age, time, and cohort effects in the past, which are reflected in the model will continue in the future that may not be the case.

Parameter estimation of each model was performed using the R package StMoMo, as briefly introduced in Section 2. Actually, models discussed in this study can be expressed as the form of generalized linear models (GLM), as discussed in Villegas *et al.* (2018), where the number of deaths

Figure 5: Estimated parameters by models (females).

by age and year as dependent variable with Poisson or binomial link function. Therefore, estimation procedures for stochastic mortality models is consistent with those for GLM. However, there is a distinction in parameter estimation of some of stochastic mortality models. Since some models can have infinitely many available solution, which is called identifiability problem, boundary condition should be specified in advance. The estimated parameters in each model are shown in Figures 4 and 5.

4.2. Comparison of models

In order to evaluate the suitability of each model considered in the previous section, goodness-of-fit, residuals, and the reasonability of generated simulation paths were compared. The goodness-of-fit was compared using Akaike information criterion (AIC) and Bayesian information criterion (BIC). The results show that models considering cohort effects tend to fit better for both genders, which is consistent with previous studies; that is, the RH, CBD-M7, and PLAT models are superior to the LC, CBD, and APC models for both genders. However, models should be evaluated by considering other aspects together with goodness-of-fit.

Residual analysis allows us to check the appropriateness of model formulation and to identify areas of improvement in the model. Deviance residuals by age, calendar year, and birth year are presented in Figure 6. For the LC and CBD models, significant patterns or heteroscedasticity were

Figure 6: Residuals plots for each model.

observed in the residuals for both genders. Those anomalies are less noticeable in the other four models, suggesting possible cohort effects. Although the residual plots for the APC, RH, CBD-M7, and PLAT models are not satisfactory, the RH and PLAT models show better results than the APC and CBD-M7 models. The results of the residual analysis are fairly consistent with the results of goodness-of-fit analysis.

Longevity risk analysis requires sample paths of future mortality rates by age to evaluate the variability of estimated annuity values affected by random movements of mortality rates. Therefore, the ability to generate reasonable sample paths of future mortality rates should be considered as one of the primary criteria to assess the suitability of a stochastic mortality model. For each of the six models, sample trajectories for selected ages were generated based on the fitted multivariate random walk with drift models for κ_t and univariate ARIMA(1, 1, 0) models for γ_{t-x} .

Figures 7 and 8 present sample paths of mortality rate generated by each model for age 65. Remarkably, many sample paths generated by the CBD-M7 and PLAT models show mortality rate increases in the future. This contradicts our expectations on mortality rates based on the historical pattern of decreasing mortality.

Although the goodness-of-fit for those models was acceptable, a number of unreasonable scenarios can be generated, which can mislead the evaluation of longevity risk. The structure of the models may not be appropriate as a relatively large number of parameters are used to fit mortality data within a short period. Also, high variability in mortality rates projected by the CBD-M7 and PLAT models implies the difficulty in finding appropriate time series models for time trend and cohort effect due

Figure 7: Simulation paths of mortality rate for age 65 (male).

Figure 8: Simulation paths of mortality rate for age 65 (female).

to relatively short period of data. However, the LC, CBD, APC, and RH models generate plausible sample paths of mortality rate. Since the variability of sample paths is different among these models, where the variability is from the number of time series models and their structure in a model, each model can be useful for longevity risk analysis depending on the outlook of the analyst.

Although direct comparison of results between this study and previous literatures based on other populations is not feasible since the range of age, gender, and year of the data do not match, there are some differences and similarities. For goodness-of-fit of models, the result from this study is similar to the result obtained from England and Wales males aged 60–89 years for 1961–2004, but is quite different from the result from US males aged 60–89 years for 1968–2003, both of which were studied in Cairns *et al.* (2009). In addition, Villegas *et al.* (2018), which explored from England and Wales males aged 55–89 years for 1961–2011, obtained similar result to this study. Another similarity between this study and Cairns *et al.* (2009), Villegas *et al.* (2018) is that the LC and CBD model showed strong pattern in residual plot. Also, the PLAT model resulted in implausible mortality forecast as found in this study.

However, the CBD-M7 model produced plausible mortality forecast while it has favorable goodness-of-fit result in Cairns *et al.* (2009) and Villegas *et al.* (2018). The APC model was considered unfavorable due to strong pattern in residual plot in Cairns *et al.* (2009) and due to decreasing variability according to age in Villegas *et al.* (2018). Comparison indicates that performance of models depends on the population. Therefore, model risk should be considered in practice.

5. Life expectancy and annuity values

Stochastic mortality models allow us to quantify the variability as well as distribution life expectancy using simulations. In terms of goodness-of-fit and the ability to generate a reasonable set of sample paths of mortality rate, the APC and RH models are relatively suitable for future mortality/longevity risk analysis. This aligns with previous studies on stochastic mortality models using Korean mortality data, which consistently suggested the APC model based on goodness-of-fit, residual analysis, and robustness, without observing the ability to generate a reasonable set of sample paths.

Using the APC and RH models, the 30-year temporary curtate life expectancy was estimated by simulation for starting ages 55, 60, and 65 in 2020. Let K_x be random variable indicating the future lifetime in complete years lived by a survivor aged x . Then, the n -year temporary curtate life expectancy is defined by the expected value of $\min(K_x, n)$. The quantity is calculated by

$$E[\min(K_x, n)] = \sum_{j=0}^{\infty} \min(j, n) \cdot \Pr[K_x = j] = \sum_{j=1}^n \Pr[K_x \geq j].$$

For comparison with a model without cohort effects, the same calculations were performed using the LC model. We set the value of n to 30 since the model was fitted using data containing ages up to 99. Since we have a two-dimensional mortality table according to age and year, the probability of death is denoted for a survivor aged x in year t by $q(x, t)$. Then the n -year temporary curtate life expectancy of a life aged x in year s is expressed as

$$\sum_{j=1}^n \left[\prod_{k=0}^{j-1} \{1 - q(x + k, t + k)\} \right],$$

where i is the annual interest rate. The estimated value, variability, and percentiles of life expectancy were observed based on 10,000 simulations and compared with the simulation results among the LC,

Figure 9: The range of mortality rates by simulation using the APC and RH models.

APC and RH models. Figure 9 shows fan charts of simulated mortality rates of ages 55, 65, and 75 for future years using the APC and RH models. The fan charts were plotted using 2.5, 10, 25, 75, 90, and 97.5 percentiles. The simulated mortality rates using the APC model are more consistent between males and females than the RH models. As discussed in Villegas *et al.* (2018), the historical volatility of mortality rates was observed to evaluate the reasonableness of fan charts. Since the historical volatility of mortality rates, shown in Figure 10, does not present any distinct increasing or decreasing pattern, there is no evidence that fan charts significantly violate historical features of mortality rates.

Estimated life expectancy, standard error, and percentiles were obtained using the mean of 10,000 simulated life expectancy values and are presented in Table 2. For a survivor aged 55, life expectancy using the LC, APC, and RH models is 26.40, 26.56, and 25.82 years respectively for males and 28.54, 28.50, and 28.87 respectively for females. Those numbers change to 23.90, 24.88, and 23.32 for males and 26.91, 27.43 and 28.23 for females at age 60, and to 20.26, 22.35, and 19.91 for males and 23.87, 25.64, and 27.05 at age 65. The comparative results among the three models for females are somewhat different than for males.

The APC and RH model tend to produce longer life expectancies than the LC model for females, while only the APC model resulted in longer life expectancies than the LC model for males, although the differences in life expectancy of males among three models are not significant when standard deviations are considered. Further, the life expectancies based on the APC model is larger than the RH model for males, but for females, the opposite result was found. The results should be carefully addressed in practice to select models when conservative mortality projection results are needed. The

Figure 10: Historical volatility of mortality rates by age.

Table 2: Simulated 30-year term life expectancy of a survivor aged 55, 60, or 65

Models		Males			Females		
		55	60	65	55	60	65
LC	Mean	26.3977	23.8996	20.2568	28.5414	26.9090	23.8690
	Standard Error	0.1158	0.1486	0.1556	0.0805	0.1314	0.1710
	Median	26.3998	23.9018	20.2583	28.5435	26.9108	23.8710
	95th percentile	26.5835	24.1396	20.5089	28.6684	27.1194	24.1454
	99th percentile	26.6561	24.236	20.6153	28.7161	27.1982	24.2535
APC	Mean	26.5628	24.8847	22.3478	28.4982	27.4326	25.6355
	Standard Error	0.1498	0.1681	0.2267	0.0832	0.1264	0.2012
	Median	26.5668	24.8899	22.3528	28.5007	27.4373	25.6424
	95th percentile	26.8041	25.1554	22.7147	28.6301	27.6332	25.9570
	99th percentile	26.8979	25.2613	22.8597	28.6786	27.7108	26.0813
RH	Mean	25.8180	23.3249	19.9086	28.8720	28.2314	27.0507
	Standard Error	0.1591	0.1611	0.1627	0.0630	0.1074	0.2060
	Median	25.8216	23.3285	19.9113	28.8745	28.2365	27.0612
	95th percentile	26.0728	23.5859	20.1731	28.9710	28.3989	27.3724
	99th percentile	26.1763	23.6896	20.2793	29.0081	28.4610	27.4892

percentiles based on each model can be used to consider longevity risk.

Longevity risk can be quantified based on the value of life annuity. The comparison of the value of life annuity among three models is expected to show similar results as survival probabilities are main component to obtain the value of life annuity. Since interest rate is used to discounting survival probabilities to determine the value of annuity, the differences in the value of life annuity will be more or less smaller than those in life expectancy depending on assumed interest rate.

The implication of the results is that the simulated paths of future mortality greatly depend on the structure of the model. Therefore, the validity of the structure of stochastic mortality models should be carefully justified with evidence based on empirical mortality data. Historical mortality data including

Figure 11: Estimated mortality rate by age based on the APC model in 2019.

ages over 80 in Korea are still not considered enough to capture the complicated structure of some stochastic mortality models. However, follow-up studies are expected to improve the reliability of the results with increased amount of data.

Finally, estimated mortality rates by age based on the APC and RH model were compared with mortality table in 2019, which was published by Statistics Korea. It is observed that mortality has improved from the previous year in advanced age. Comparing with Figure 2 regarding recent change in mortality, the degree of mortality improvement does not seem to be consistent by year. As shown in Figures 11 and 12, both of the APC and RH model show underestimation of mortality rates in ages over 80. It can be interpreted that models tend to reflect the degree of mortality improvement for last several decades corresponding to the period of data, even though mortality improvement become smaller in recent few years.

Statistics Korea provides projected mortality rates up to 2067 that were derived from various demographic assumptions on population changes and the Li-Lee-Gerland model which is an extension of the LC model. As presented in Figures 11 and 12, the projected mortality rates in 2019 overestimated the actual mortality. Therefore, stochastic mortality models such as the APC and RH model are more suitable to accommodate longevity risk. Further, underestimated mortality rates can be used as a conservative mortality assumption for annuity pricing in case of adverse movement in mortality in the future.

6. Concluding remarks

To ensure an appropriate retirement income, it is important to maintain sustainability of the financial soundness of pension programs, portfolios of individual annuities, and reverse mortgages. Since

Figure 12: Estimated mortality rate by age based on the RH model in 2019.

mortality assumptions are directly related to the amount of pension and annuity payments as well as the expected length of payment, the estimation and associated risk inherent in future mortality rates should be evaluated with a model reflecting the characteristics of human mortality. Various stochastic mortality models have been suggested to project mortality rates in the future. Typically, the mortality models accommodate age effects, time trends, and cohort effects.

The appropriateness of a certain mortality model should be tested with various aspects based on empirical data. Former case studies based on Korean mortality data compared stochastic mortality models using criteria such as goodness-of-fit, residual analysis, robustness, and prediction power using test data. However, the models were not evaluated on whether they generate reasonable trajectories of mortality rates in the future, which is the most essential element of evaluating uncertainty of the projected mortality in longevity risk management.

This study followed up on the previous case studies using updated Korean mortality data and the recently developed R package StMoMo, facilitating analysis based on popular stochastic mortality models. Focusing on the retirement ages between 55 and 99 years, separated by gender, the study investigated six mortality models in terms of goodness-of-fit, residual patterns, and ability to generate reasonable sample paths of future mortality. Further, the selected models were used to calculate life expectancy and the value of life annuity to understand how estimated results can be different due to uncertainty in the mortality projection and how this uncertainty is different across the models.

Importantly, it was observed that goodness-of-fit does not guarantee the desired ability to generate reasonable sample paths of future mortality. This result implies that the variability of mortality projection results and the reasonableness of simulated future mortality rates of each model should be carefully observed and reflected as a criterion of model selection. Among the six models considered in this study, the Age-Period-Cohort (APC) and Renshaw-Haberman (RH) models were found to be rel-

atively more appropriate for longevity risk analysis. Models without cohort effects did not fit the data well. Although the CBD-M7 and PLAT models fits the data well, generated sample paths of future mortality rates were inappropriate. The result of model selection is consistent with the conclusions of previous studies, even though they did not observe the reasonability of generated sample paths of mortality rates. However, the models should be evaluated with follow-up studies since each model is expected to capture the complicated characteristics of human mortality with more abundant mortality data. In addition, compared to previous tools, the R package StMoMo was found to be a more flexible and comprehensive tool that allow various analyses regarding stochastic mortality models.

The availability of mortality data with extended age ranges and periods in the future will improve the study of stochastic mortality models. Various case studies using sub-population mortality data collected from a portfolio managed by an entity will provide insights on the characteristics of mortality of the population under consideration. Further, the application of stochastic models for analyzing the mortality rate due to a specific cause or morbidity rate is a possible area of future research.

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